

Local Symmetry Preserving Operations on Polyhedra

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Universiteit Gent

Departement Toegepaste Wiskunde, Informatica en Statistiek

Openbare verdediging



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Lokale Symmetriebewarende Operaties op Veelvlakken

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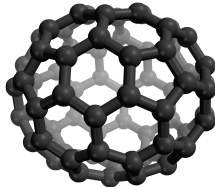
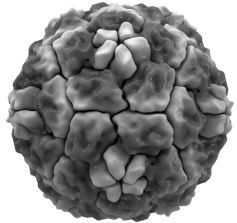
Inhoud

- ▶ Veelvlakken
- ▶ Grafen
- ▶ LSP operaties
- ▶ Constructie van LSP operaties
- ▶ LOPSP operaties
- ▶ Constructie van LOPSP operaties
- ▶ Open problemen

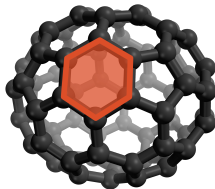
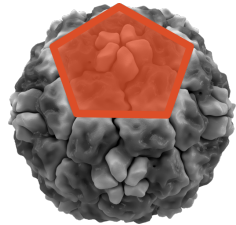
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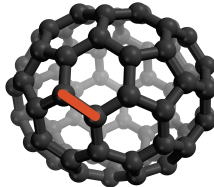
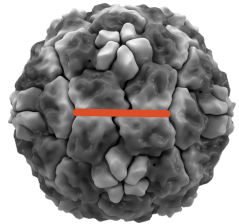
Wat zijn convexe veelvlakken?



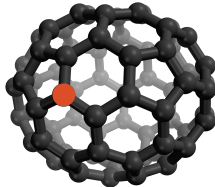
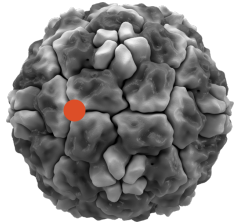
Wat zijn convexe veelvlakken?



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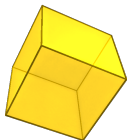


Regelmatige veelvlakken

Plato (427–347 v.C.)



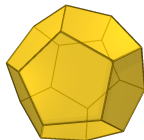
tetraëder



kubus



octaëder



dodecaëder



icosaëder

Halfregelmatige veelvlakken

Archimedes (287–212 v.C.)



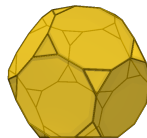
afgeknotte
tetraëder



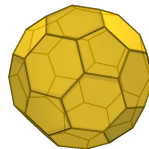
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afgeknotte
octaëder



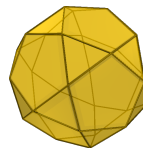
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afgeknotte
icosaëder



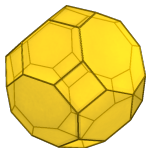
kuboctaëder



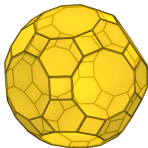
icosidodecaëder

Halfregelmatige veelvlakken

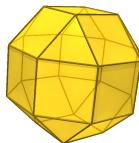
Archimedes (287–212 v.C.)



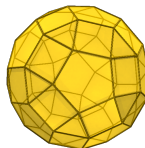
afgeknotte
kuboctaëder



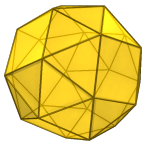
afgeknotte
icosidodecaëder



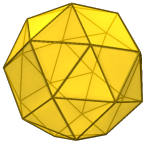
romboëdrisch
kuboctaëder



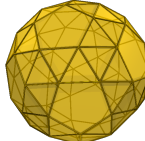
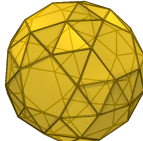
rombische
icosidodecaëder



stompe kubus



stompe dodecaëder



Operaties op regelmatige veelvlakken

Johannes Kepler (1571–1630)

62 DE FIGURARUM HARMON:

Cum enim misceantur in hoc gradu figurae diversae, quatuor propositi XXI. misceantur aut duarum aut trium simplicium figurae. Quod si duarum, tunc inter eas vel sunt Triangula vel non sunt.

Igitur Triangula sunt solidiora, quibus quidem des. IX. comparantur. Nam illa triangula sunt tria in quibus planum anguli clauduntur, cum uno Tetragonico plano anguli, item duo, quibus tria plana Triagonica cum duobus Tetragonico, uno Triagonico; quia in primo casu unus planus Tetragonico est, si quidamlibet anguli solidi sunt descripti in secundo duo soli Tetragoni, in tertio duo soli Trianguli, quae p. 2. sunt imperficula cognoscitur. Quibus itaque modis, in quibus angulum solidum clauduntur. Item, Triagonum quatuor Triagonici est, nam Tetragonum, sunt cum minoris a rebus. Congruunt igitur sex Tetragoni et Triagonum duo (id est 20 et 12.) Triagonici si figura Triacantha debet, quod appello Cubum simplex. Hic in schemate sequenti patet si Numerus 12.

62
I. Cubus simplex



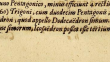
II. Rhombus



III. Rhombus



IV. Rhombus



V. Rhombus



VI. Rhombus

VII. Rhombus

VIII. Rhombus

IX. Rhombus

X. Rhombus

XI. Rhombus

XII. Rhombus

XIII. Rhombus

XIV. Rhombus

XV. Rhombus

XVI. Rhombus

XVII. Rhombus

XVIII. Rhombus

XIX. Rhombus

XX. Rhombus

XXI. Rhombus

XXII. Rhombus

XXIII. Rhombus

XXIV. Rhombus

XXV. Rhombus

XXVI. Rhombus

XXVII. Rhombus

XXVIII. Rhombus

XXIX. Rhombus

XXX. Rhombus

XXXI. Rhombus

XXXII. Rhombus

XXXIII. Rhombus

XXXIV. Rhombus

XXXV. Rhombus

XXXVI. Rhombus

XXXVII. Rhombus

XXXVIII. Rhombus

XXXIX. Rhombus

XL. Rhombus

XLI. Rhombus

XLII. Rhombus

XLIII. Rhombus

XLIV. Rhombus

XLV. Rhombus

XLVI. Rhombus

XLVII. Rhombus

XLVIII. Rhombus

XLIX. Rhombus

L. Rhombus

LI. Rhombus

LII. Rhombus

LIII. Rhombus

LIV. Rhombus

LV. Rhombus

LVI. Rhombus

LVII. Rhombus

LVIII. Rhombus

LIX. Rhombus

LX. Rhombus

LXI. Rhombus

LXII. Rhombus

LXIII. Rhombus

LXIV. Rhombus

LXV. Rhombus

LXVI. Rhombus

LXVII. Rhombus

LXVIII. Rhombus

LXIX. Rhombus

LXX. Rhombus

LXXI. Rhombus

LXXII. Rhombus

LXXIII. Rhombus

LXXIV. Rhombus

LXXV. Rhombus

LXXVI. Rhombus

LXXVII. Rhombus

LXXVIII. Rhombus

LXXIX. Rhombus

LXXX. Rhombus

LXXXI. Rhombus

LXXXII. Rhombus

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LXXXV. Rhombus

LXXXVI. Rhombus

LXXXVII. Rhombus

LXXXVIII. Rhombus

LXXXIX. Rhombus

LXXXX. Rhombus

LXXXXI. Rhombus

LXXXXII. Rhombus

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LXXXXVIII. Rhombus

LXXXXIX. Rhombus

LXXXXX. Rhombus

LXXXXXI. Rhombus

LXXXXXII. Rhombus

LXXXXXIII. Rhombus

LXXXXXIV. Rhombus

LXXXXXV. Rhombus

LXXXXXVI. Rhombus

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LXXXXXVIII. Rhombus

LXXXXXIX. Rhombus

LXXXXXX. Rhombus

LXXXXXXI. Rhombus

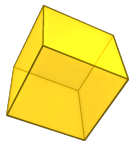
LXXXXXXII. Rhombus

LXXXXXXIII. Rhombus

LXXXXXXIV. Rhombus

Symmetriebewarende operaties op veelvlakken

John Conway (1937–2020)



seed



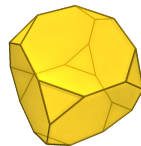
dual



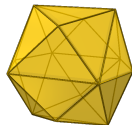
ambo



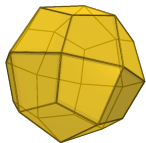
join



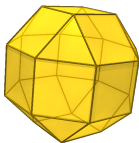
truncate



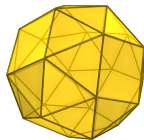
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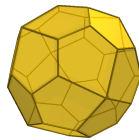
ortho



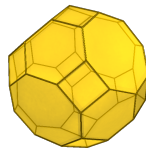
expand



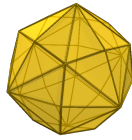
snub



gyro

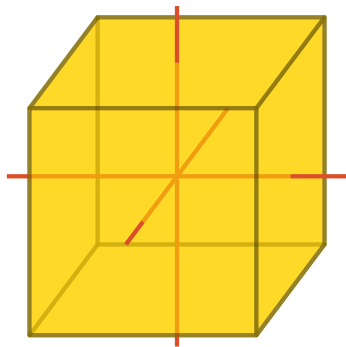


bevel

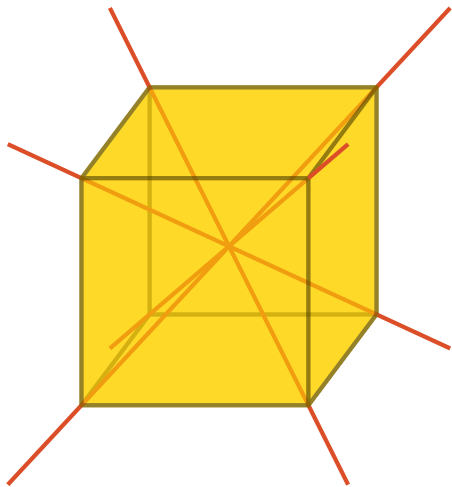


meta

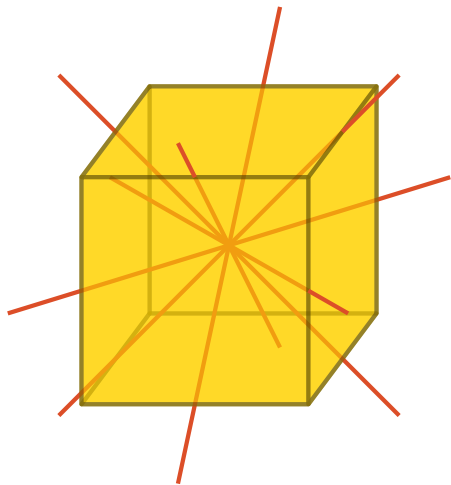
Symmetrie



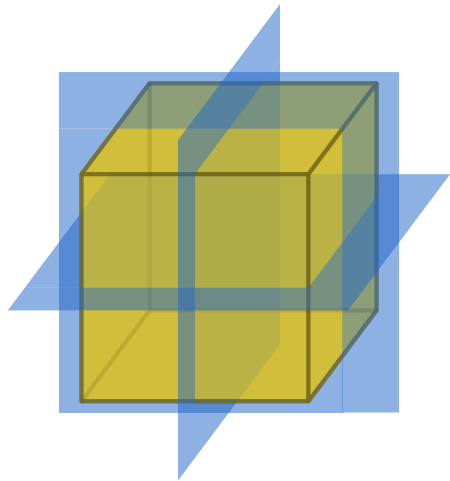
Symmetrie



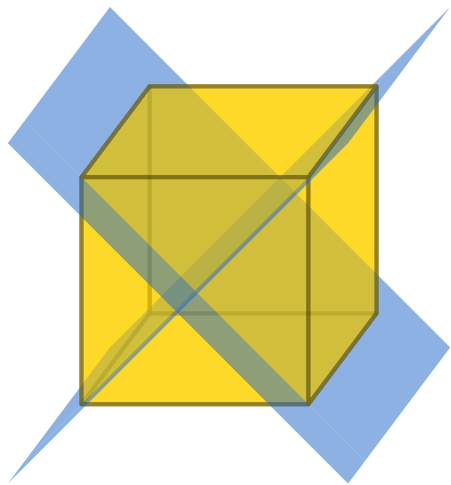
Symmetrie



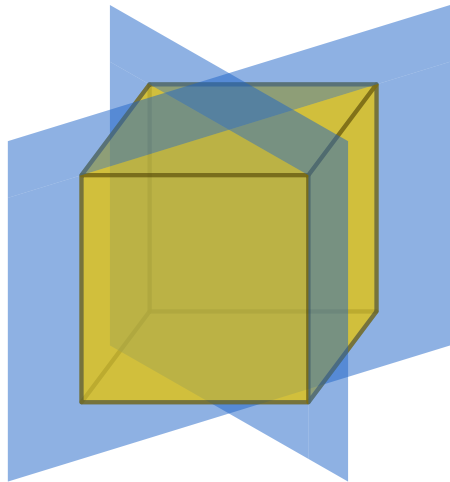
Symmetrie



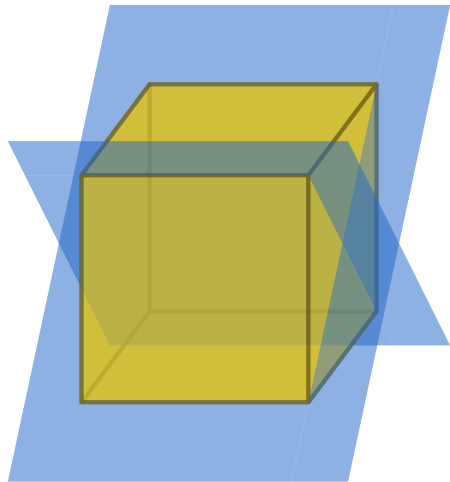
Symmetrie



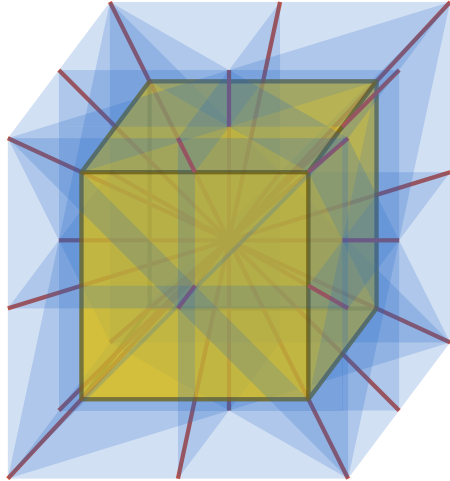
Symmetrie



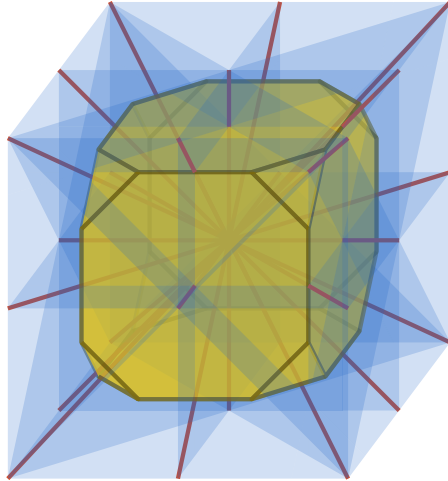
Symmetrie



Symmetrie

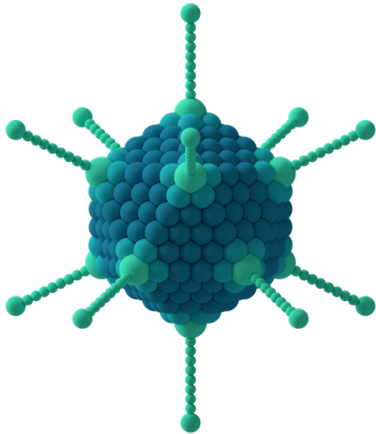


Symmetrie



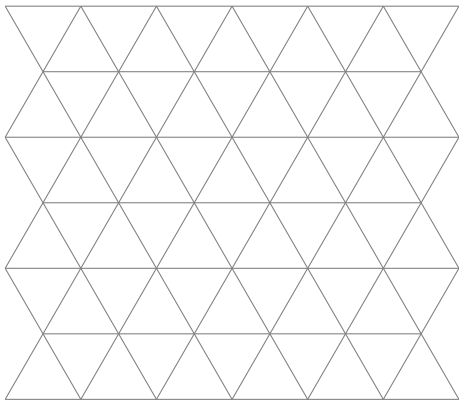
Goldberg-Coxeter constructie

Caspar en Klug (1962)



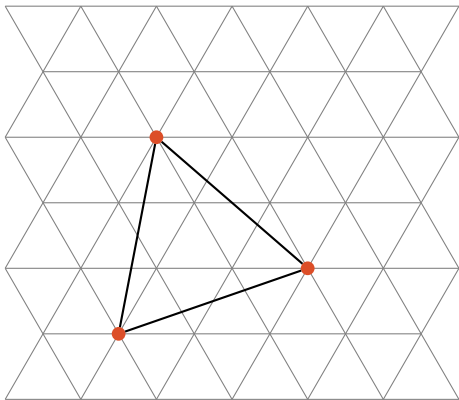
Goldberg-Coxeter constructie

Caspar en Klug (1962)



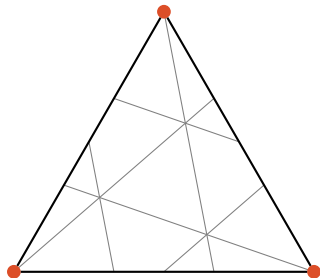
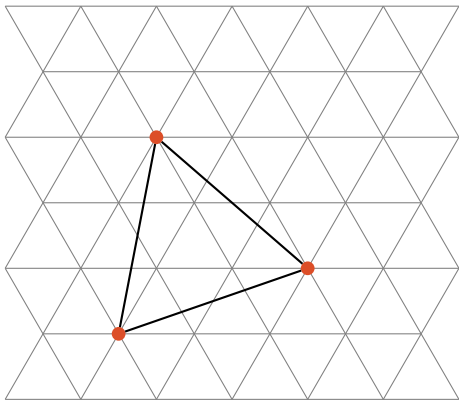
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Caspar en Klug (1962)



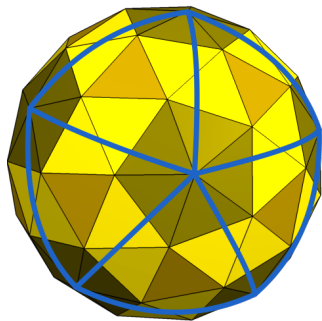
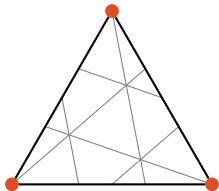
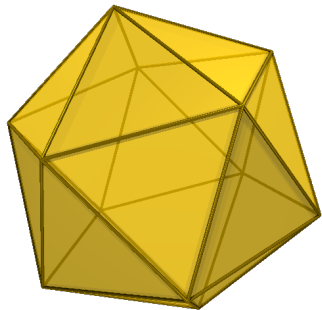
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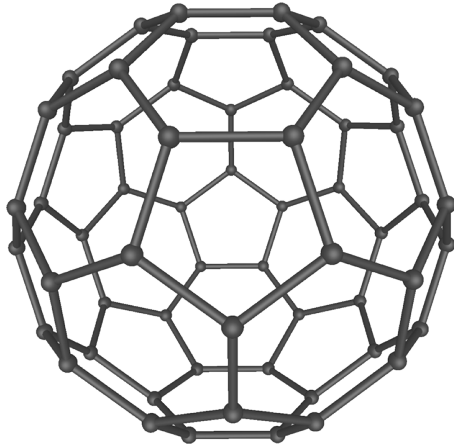
Goldberg-Coxeter constructie

Caspar en Klug (1962)



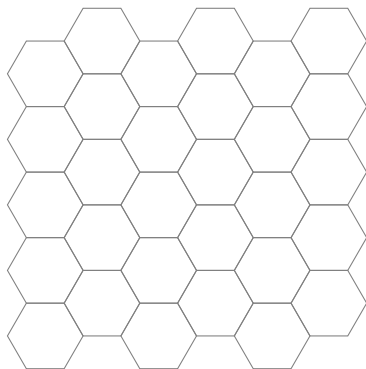
Goldberg-Coxeter constructie

Michael Goldberg (1937)



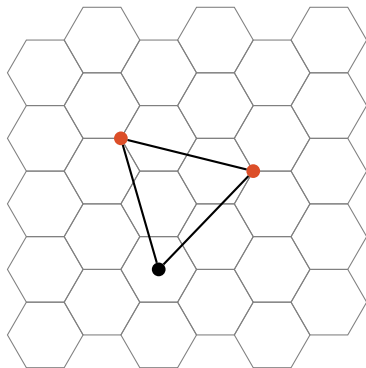
Goldberg-Coxeter constructie

Michael Goldberg (1937)



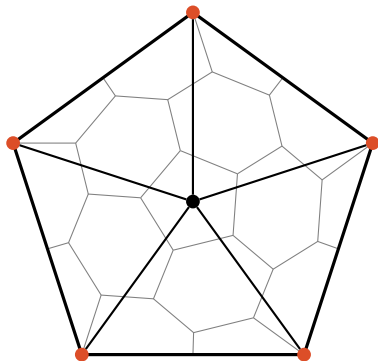
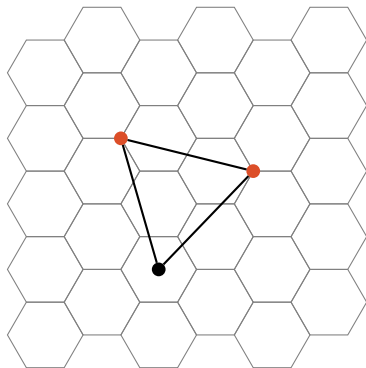
Goldberg-Coxeter constructie

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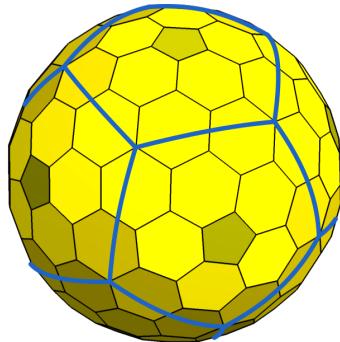
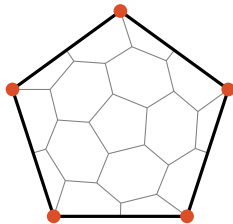
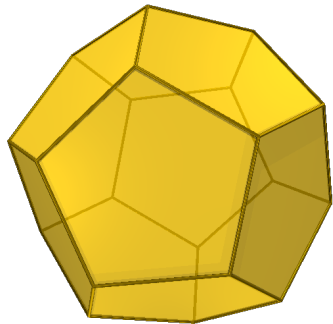
Goldberg-Coxeter constructie

Michael Goldberg (1937)



Goldberg-Coxeter constructie

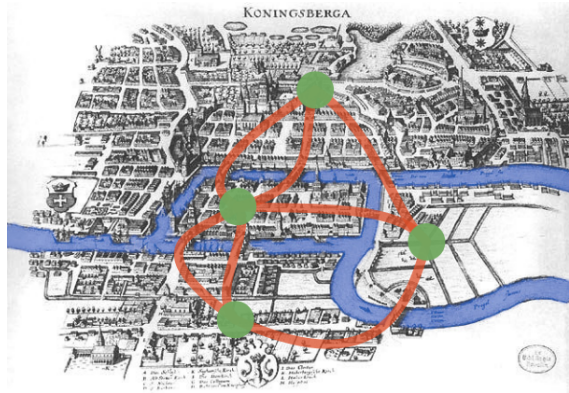
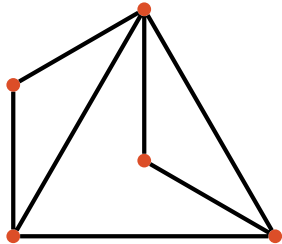
Michael Goldberg (1937)



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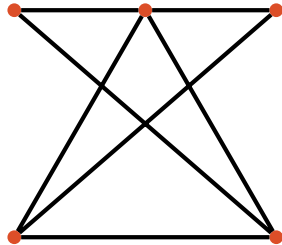
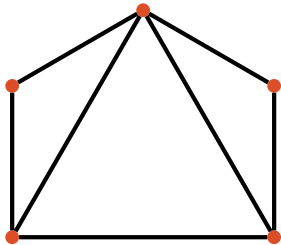
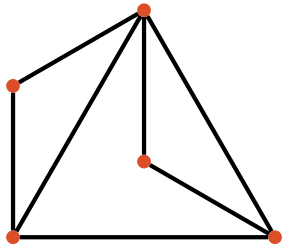
Wat is een graaf?



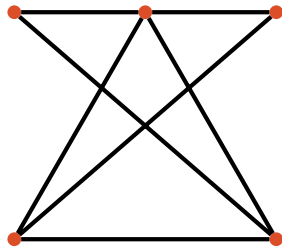
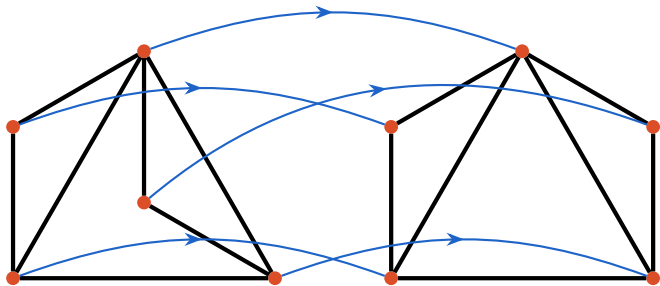
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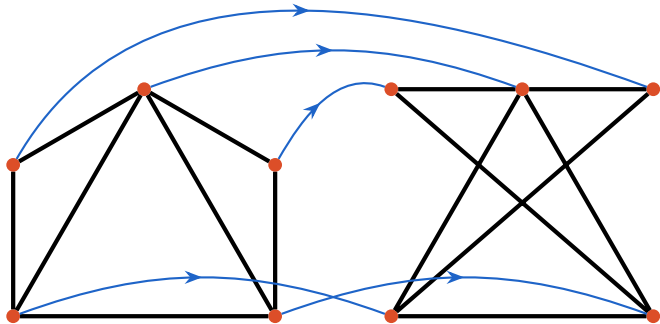
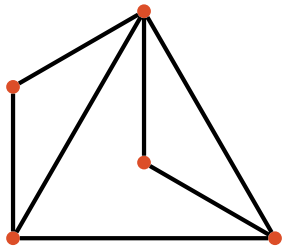
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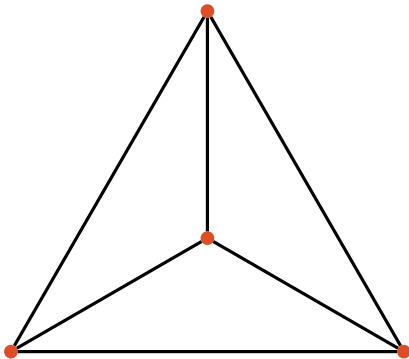
Wat is een graaf?



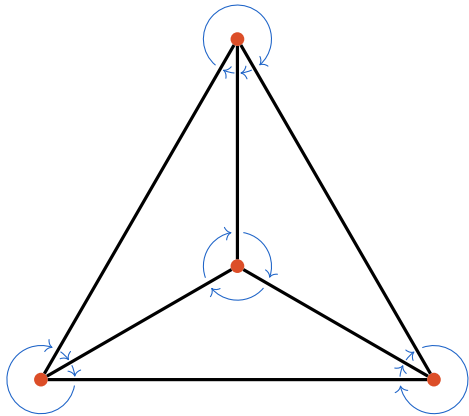
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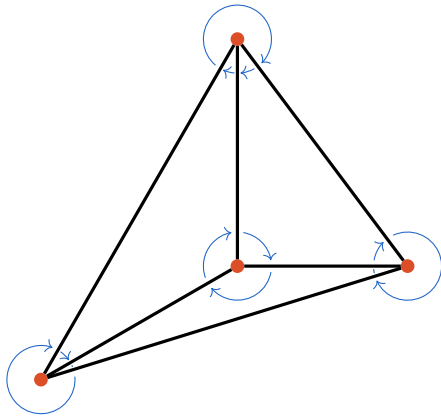
Vlakke grafen



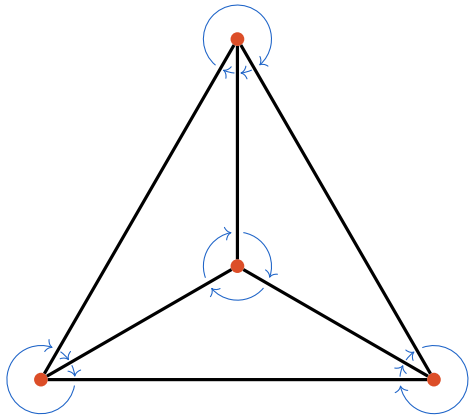
Vlakke grafen



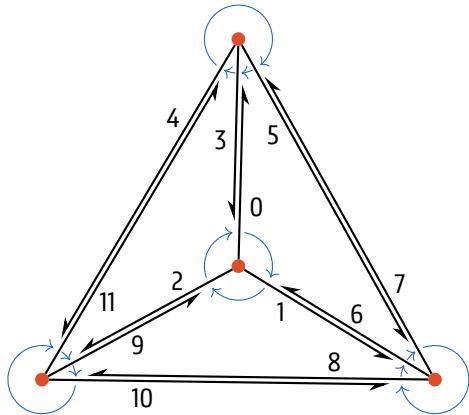
Vlakke grafen



Vlakke grafen



Vlakke grafen



$$E = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$$

$$\sigma(0) = 1$$

$$\theta(0) = 3$$

$$\sigma(1) = 2$$

$$\theta(1) = 6$$

$$\sigma(2) = 0$$

$$\theta(2) = 9$$

$$\sigma(3) = 4$$

$$\theta(3) = 0$$

$$\sigma(4) = 5$$

$$\theta(4) = 11$$

$$\sigma(5) = 3$$

$$\theta(5) = 7$$

$$\sigma(6) = 7$$

$$\theta(6) = 1$$

$$\sigma(7) = 8$$

$$\theta(7) = 5$$

$$\sigma(8) = 6$$

$$\theta(8) = 10$$

$$\sigma(9) = 10$$

$$\theta(9) = 2$$

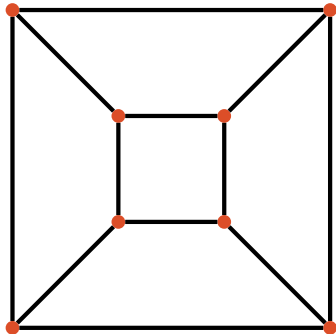
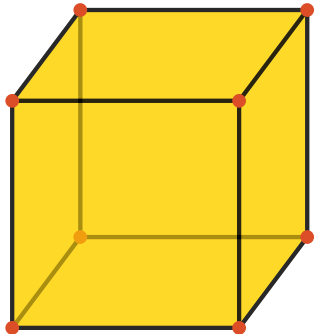
$$\sigma(10) = 11$$

$$\theta(10) = 8$$

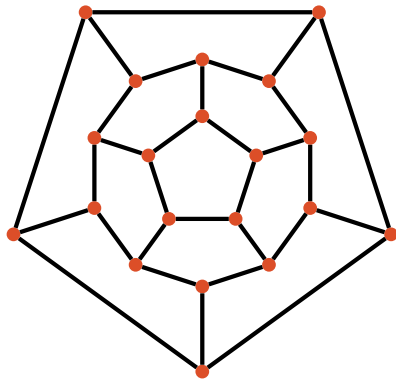
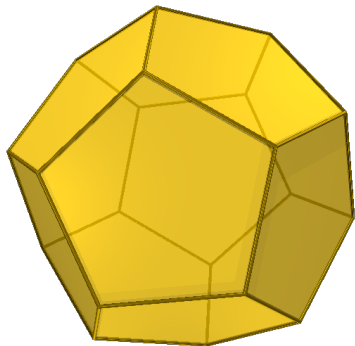
$$\sigma(11) = 9$$

$$\theta(11) = 4$$

Polyëdrische grafen



Polyëdrische grafen

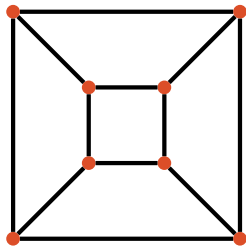
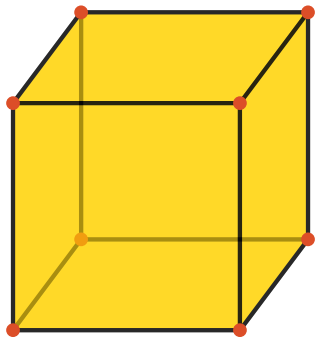


Polyëdrische grafen

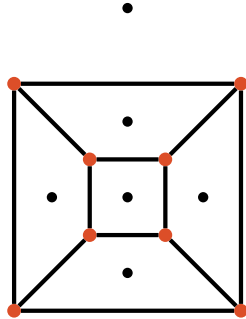
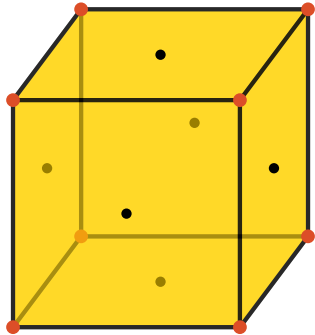
Stelling van Steinitz

De toppen en bogen van een convex veelvlak vormen een driesamenhangende vlakke graaf, en elke driesamenhangende vlakke graaf kan gerealiseerd worden door een convex veelvlak.

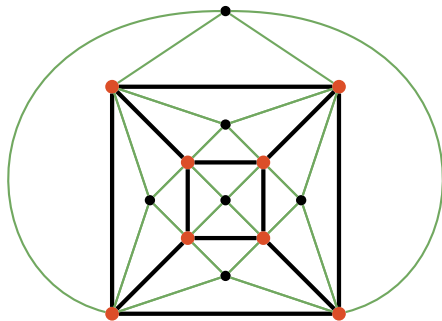
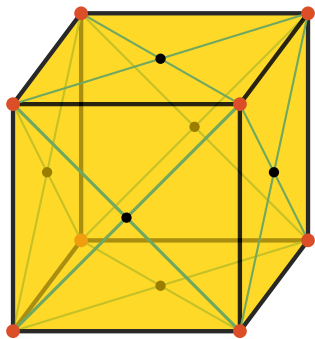
Kamersystemen



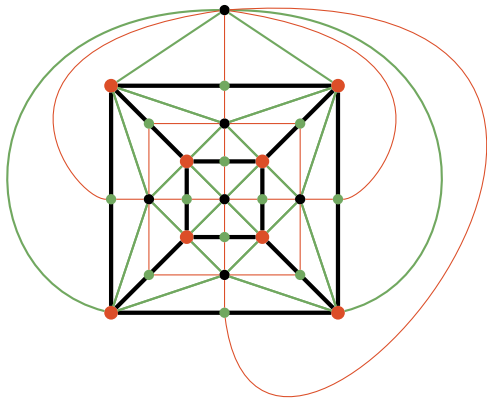
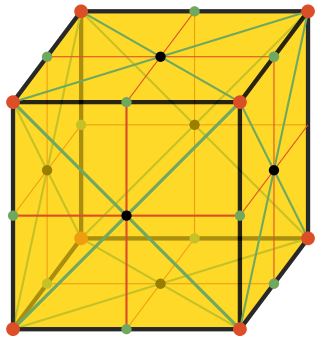
Kamersystemen



Kamersystemen



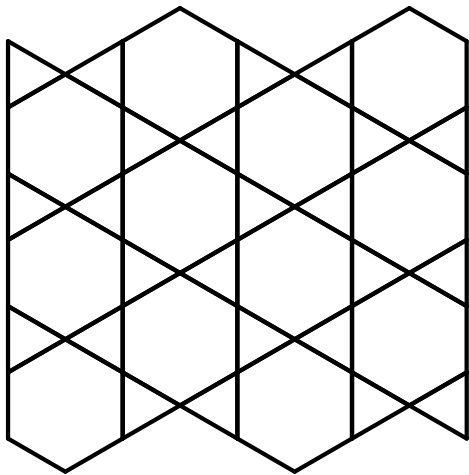
Kamersystemen



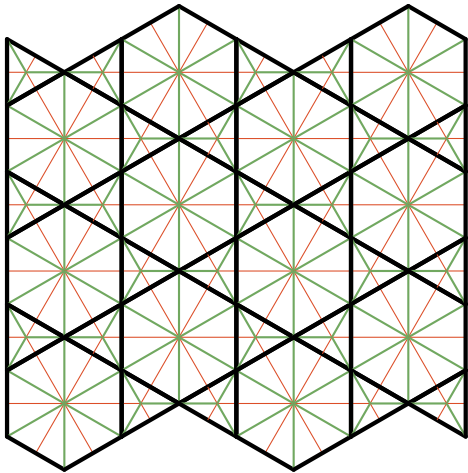
Inhoud

- ▶ Veelvlakken
- ▶ Grafen
- ▶ LSP operaties
- ▶ Constructie van LSP operaties
- ▶ LOPSP operaties
- ▶ Constructie van LOPSP operaties
- ▶ Open problemen

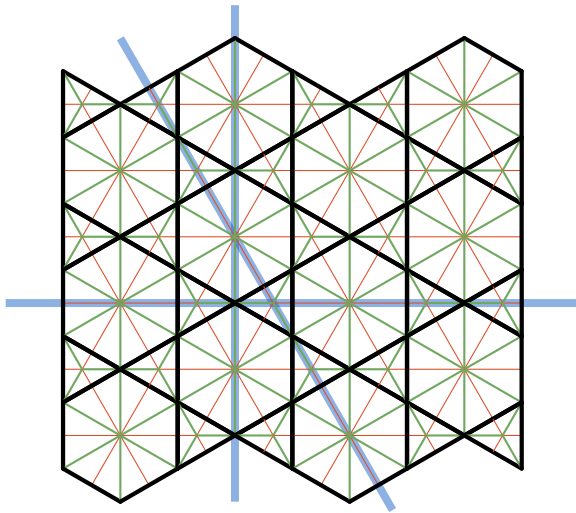
Lokale symmetriebewarende operaties



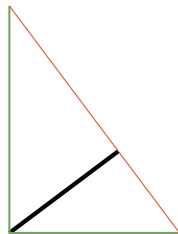
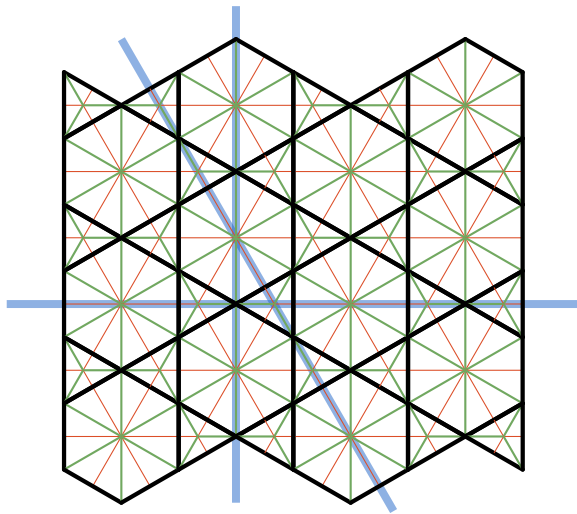
Lokale symmetriebewarende operaties



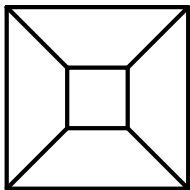
Lokale symmetriebewarende operaties



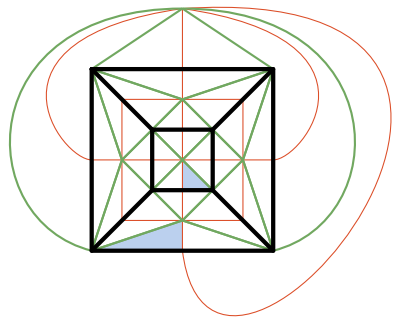
Lokale symmetriebewarende operaties



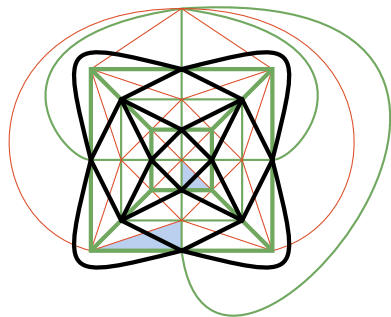
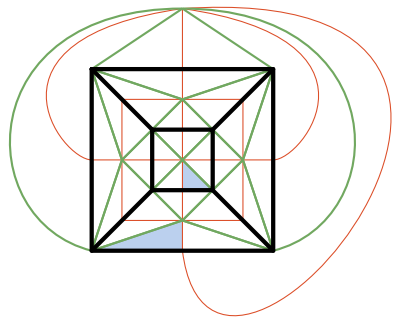
Lokale symmetriebewarende operaties



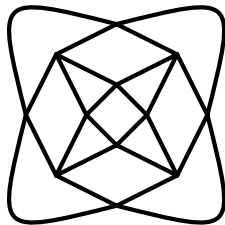
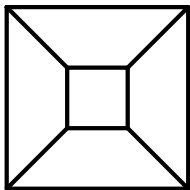
Lokale symmetriebewarende operaties



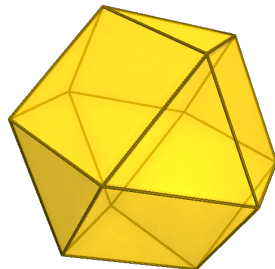
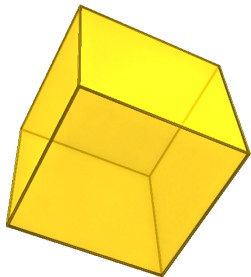
Lokale symmetriebewarende operaties



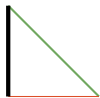
Lokale symmetriebewarende operaties



Lokale symmetriebewarende operaties



Conway operations



dual



ambo



join



truncate



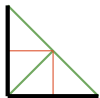
zip



needle



kis



ortho



expand



chamfer



subdivide



loft



meta



bevel



quinto



join-lace



lace



stake



medial

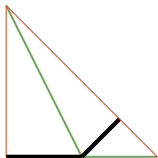
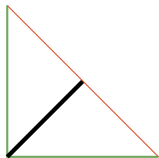


join-kis-kis

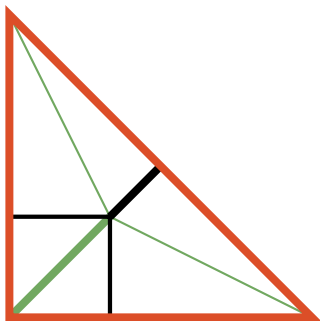


cross

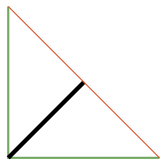
Samenstelling



=

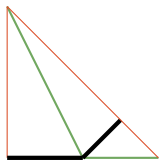


Inflatiefactor



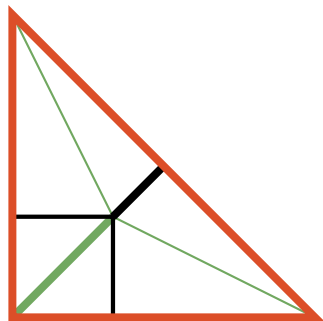
2

×



3

=



6

=

Inhoud

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Constructie

r	k = 1	k = 2	k = 2
1	2	2	2
2	2	2	2
3	4	4	4
4	6	6	6
5	6	6	4
6	20	20	20
7	28	28	20
8	58	58	54
9	82	82	64
10	170	168	144
11	204	200	132
12	496	492	404
13	650	640	396
14	1432	1400	1112
15	1824	1786	1100
16	4114	3952	2958

17	5078	4900	2769
18	11874	11150	7972
19	14808	14058	7560
20	33978	30998	21300
21	41794	38964	20076
22	97096	85976	56296
23	118572	107784	52380
24	277208	237482	148956
25	337216	298546	138384
26	788342	652236	392096
27	953060	820960	362499
28	2239396	1786222	1027488
29	2697088	2250816	945612
30	6350014	4875076	2687408
31	7618068	6153604	2466156
32	17972390	13262574	7007118
33	21487746	16773086	6409664
34	50805716	35985748	18222032

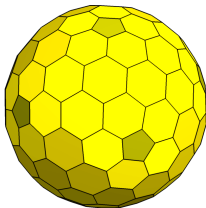
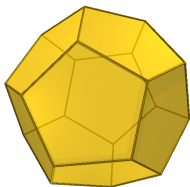
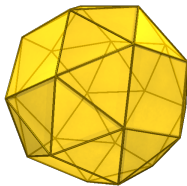
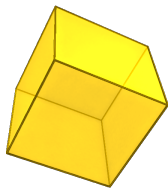
Constructie

35	60573248	45592594	16623268
36	143425040	97394726	47287986
37	170530518	123628298	43038260
38	404413576	262983002	122451618
39	479711448	334473144	111200316
40	1139138344	708583784	316474370
41	1348351130	902941632	286833388
42	3205480518	1905439964	816499872
43	3786676452	2432756832	738710312
44	9012067074	5114410850	2103076126
45	10627114786	6542306684	1899585208
46	25315977970	13703891826	5408673658
47	29803635416	17563047708	4878021416
48	71060710972	36659785890	13890262050
49	83530248078	47071210950	12510181242
50	199322142978	97920815584	35624781874
:	:	:	:

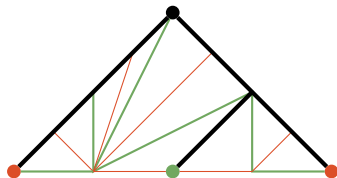
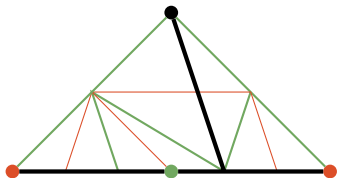
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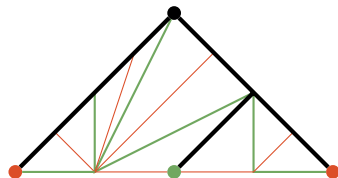
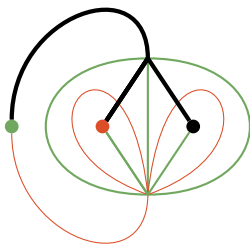
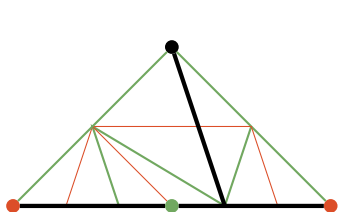
Chirale operaties



Lokale oriëntatiebewarende symmetriebewarende operaties



Lokale oriëntatiebewarende symmetriebewarende operaties



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